

Maths with Melissa

A-Level Polynomials Practice

Factor Theorem • Cubic Graphs • Discriminants

Name: _____

Date: _____

Instructions: Show all working clearly. For graph questions, mark and label all x- and y-intercepts. Worked solutions are on the final pages.

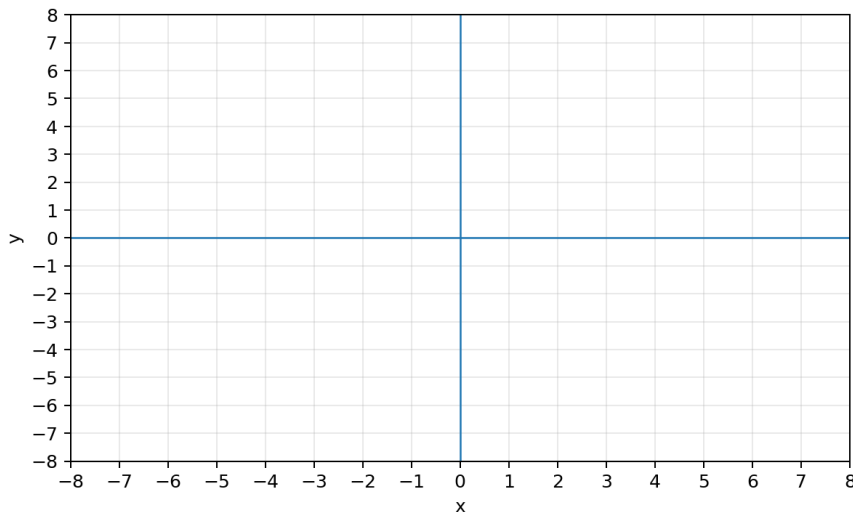
Practice Set 1

Total: 9 marks

(a) The polynomial $x^3 - 6x^2 + 3x + 10$ has a factor $(x - a)$, where a is a positive integer in the interval $0 < a < 5$. Find the value of a , showing your working clearly. **[2 marks]**

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(b) Sketch the graph of $y = x^3 - 6x^2 + 3x + 10$, clearly showing the coordinates of the points at which the graph intersects both the x- and y-axes. **[4 marks]**



(c) A different polynomial can be factorised to give $(x + 3)(x^2 - kx + 9)$. Given that $x = -3$ is the only real root of the polynomial, find the possible values of k . **[3 marks]**

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Practice Set 2

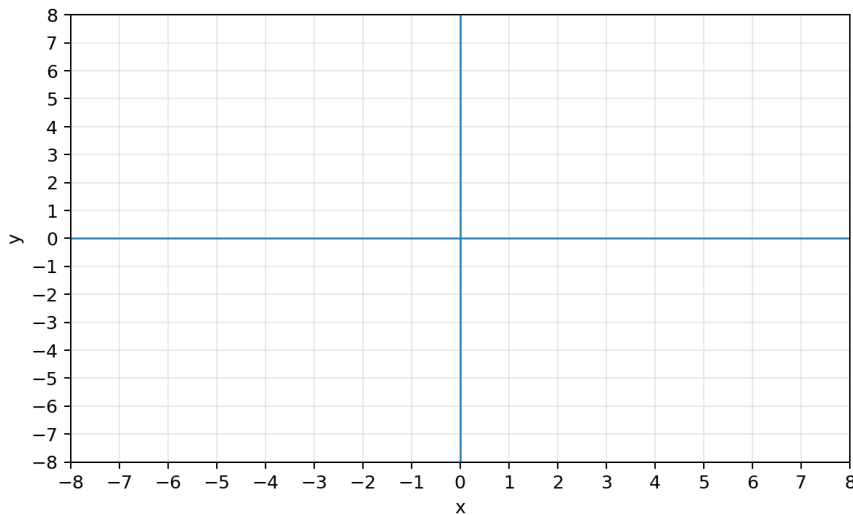
Total: 9 marks

(a) The polynomial $x^3 - 9x^2 + 6x + 56$ has a factor $(x - a)$, where a is a positive integer in the interval $0 < a < 5$. Find the value of a , showing your working clearly. **[2 marks]**

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(b) Sketch the graph of $y = x^3 - 9x^2 + 6x + 56$, clearly showing the coordinates of the points at which the graph intersects both the x - and y -axes. **[4 marks]**



(c) A different polynomial can be factorised to give $(x + 1)(x^2 - kx + 1)$. Given that $x = -1$ is the only real root of the polynomial, find the possible values of k . **[3 marks]**

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Worked Solutions - Practice Set 1

(a)

Let $f(x) = x^3 - 6x^2 + 3x + 10$.

By the Factor Theorem, if $(x - a)$ is a factor then $f(a) = 0$.

Test $x = 2$: $f(2) = 8 - 24 + 6 + 10 = 0$.

Therefore $a = 2$.

(b)

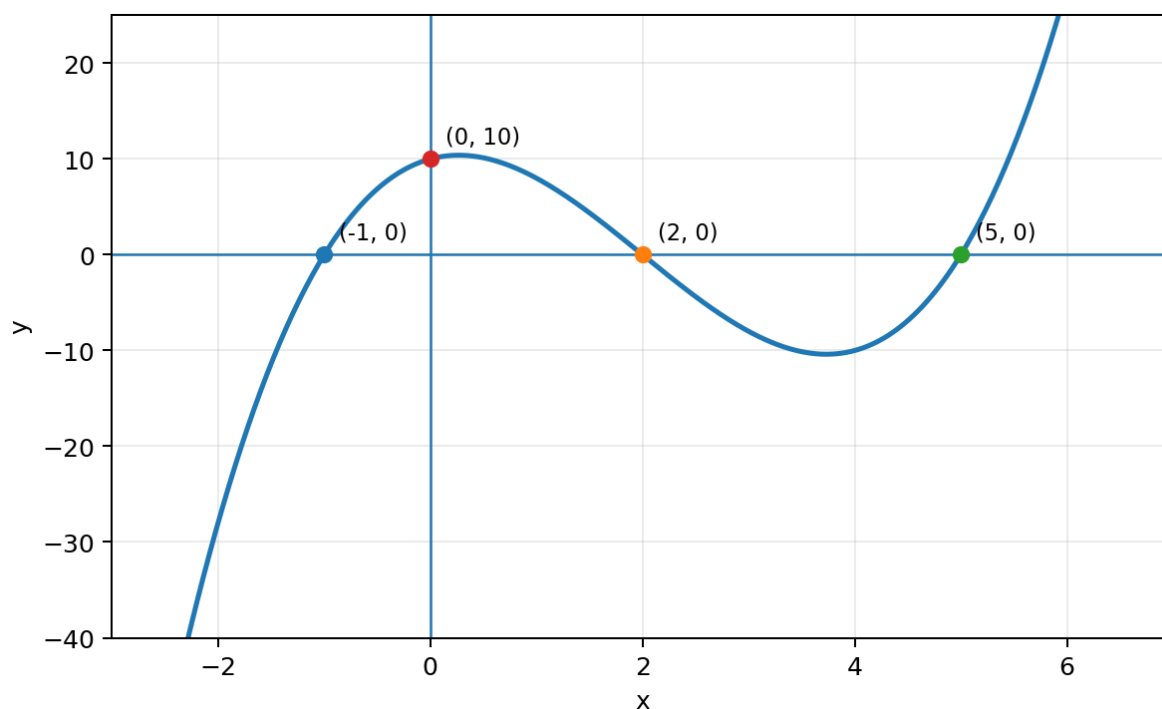
Divide by $(x - 2)$: $x^3 - 6x^2 + 3x + 10 = (x - 2)(x^2 - 4x - 5)$.

Factorise the quadratic: $x^2 - 4x - 5 = (x - 5)(x + 1)$.

So $y = (x - 2)(x - 5)(x + 1)$.

x-intercepts: $(-1, 0)$, $(2, 0)$, $(5, 0)$.

y-intercept: when $x = 0$, $y = 10$, so $(0, 10)$.



(c)

For $x^2 - kx + 9$, the discriminant is $k^2 - 36$.

For no extra real roots, $k^2 - 36 < 0$, so $-6 < k < 6$.

Check $k = -6$: $x^2 + 6x + 9 = (x + 3)^2$, so the only distinct real root is still $x = -3$.

Check $k = 6$: $x^2 - 6x + 9 = (x - 3)^2$, which introduces $x = 3$.

Therefore $-6 \leq k < 6$.

Worked Solutions - Practice Set 2

(a)

Let $f(x) = x^3 - 9x^2 + 6x + 56$.

By the Factor Theorem, if $(x - a)$ is a factor then $f(a) = 0$.

Test $x = 4$: $f(4) = 64 - 144 + 24 + 56 = 0$.

Therefore $a = 4$.

(b)

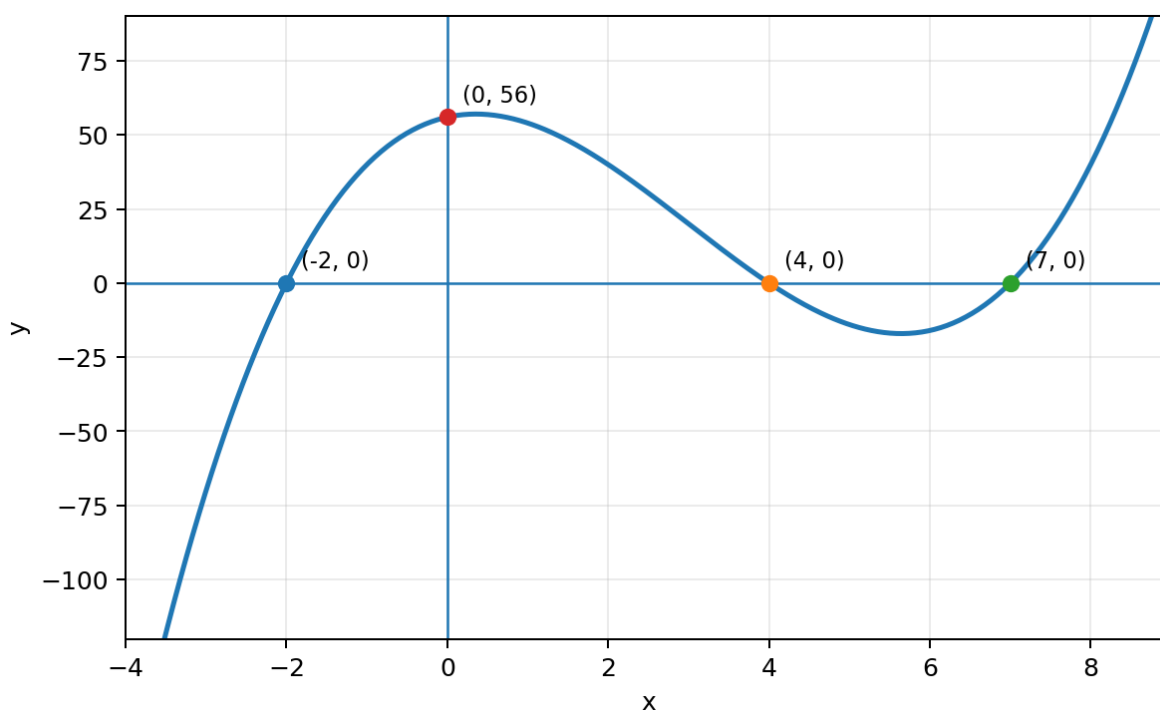
Divide by $(x - 4)$: $x^3 - 9x^2 + 6x + 56 = (x - 4)(x^2 - 5x - 14)$.

Factorise the quadratic: $x^2 - 5x - 14 = (x - 7)(x + 2)$.

So $y = (x - 4)(x - 7)(x + 2)$.

x-intercepts: $(-2, 0)$, $(4, 0)$, $(7, 0)$.

y-intercept: when $x = 0$, $y = 56$, so $(0, 56)$.



(c)

For $x^2 - kx + 1$, the discriminant is $k^2 - 4$.

For no extra real roots, $k^2 - 4 < 0$, so $-2 < k < 2$.

Check $k = -2$: $x^2 + 2x + 1 = (x + 1)^2$, so the only distinct real root is still $x = -1$.

Check $k = 2$: $x^2 - 2x + 1 = (x - 1)^2$, which introduces $x = 1$.

Therefore $-2 \leq k < 2$.

Quick Answers

	(a)	(b) intercepts	(c)
Set 1	$a = 2$	x: $(-1,0)$, $(2,0)$, $(5,0)$; y: $(0,10)$	$-6 \leq k < 6$
Set 2	$a = 4$	x: $(-2,0)$, $(4,0)$, $(7,0)$; y: $(0,56)$	$-2 \leq k < 2$